

Mark Scheme (Results)

Summer 2013

GCE Further Pure Mathematics 3 (6669/01R)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
 - M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
 - A marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
 - B marks are unconditional accuracy marks (independent of M marks)
 - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
 - ft – follow through
 - the symbol \surd will be used for correct ft
 - cao – correct answer only
 - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
 - isw – ignore subsequent working
 - awrt – answers which round to
 - SC: special case
 - oe – or equivalent (and appropriate)
 - dep – dependent
 - indep – independent
 - dp decimal places
 - sf significant figures
 - * The answer is printed on the paper
 - \square The second mark is dependent on gaining the first mark
4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
 7. Ignore wrong working or incorrect statements following a correct answer.
 8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to $x =$

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to $x =$

2. Formula

Attempt to use correct formula (with values for a , b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

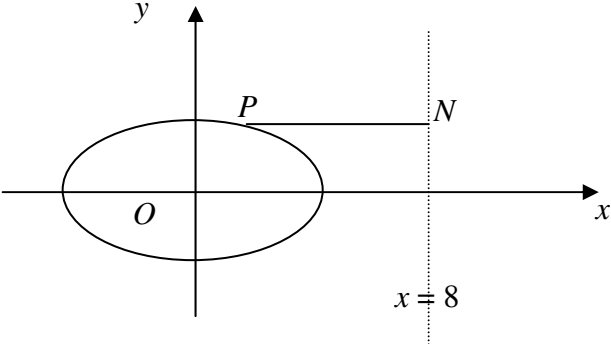
Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme		Marks
	Foci ($\pm 5, 0$), Directrices $x = \pm \frac{9}{5}$		
1.	$(\pm)ae = (\pm)5$ and $(\pm)\frac{a}{e} = (\pm)\frac{9}{5}$	Correct equations (ignore \pm 's)	B1
	so $e = \frac{5}{a} \Rightarrow \frac{a^2}{5} = \frac{9}{5} \Rightarrow a^2 = 9$ or $a = \frac{5}{e} \Rightarrow \frac{5}{e^2} = \frac{9}{5} \Rightarrow e = \frac{5}{3} \Rightarrow a = 3$	M1: Solves using an appropriate method to find a^2 or a	M1A1
		A1: $a^2 = 9$ or $a = (\pm)3$	
	$b^2 = a^2e^2 - a^2 \Rightarrow b^2 = 25 - 9$ so $b^2 = 16 \quad (\Rightarrow b = 4)$ or $b^2 = a^2(e^2 - 1) \Rightarrow b^2 = 9\left(\frac{25}{9} - 1\right)$ $b^2 = 16 \quad (\Rightarrow b = 4)$	M1: Use of $b^2 = a^2(e^2 - 1)$ to obtain a numerical value for b^2 or b	M1 A1
	A1: $b^2 = 16$ or $b = (\pm)4$		
	So $\frac{x^2}{9} - \frac{y^2}{16} = 1$	M1: Use of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with their a^2 and b^2	M1 A1
		A1: Correct hyperbola in any form.	
			(7)

Question Number	Scheme		Marks
2.	$l_1: (\mathbf{i} - \mathbf{j} + \mathbf{k}) + \lambda(4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ $l_2: (3\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}) + \lambda(-4\mathbf{i} + 6\mathbf{j} + \mathbf{k})$		
(a)	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 3 & 2 \\ -4 & 6 & 1 \end{vmatrix} = -9\mathbf{i} - 12\mathbf{j} + 36\mathbf{k}$	M1: Correct attempt at a vector product between $4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ and $-4\mathbf{i} + 6\mathbf{j} + \mathbf{k}$ (if the method is unclear then 2 components must be correct) allowing for the sign error in the y component.	M1A1
		A1: Any multiple of $(3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k})$	
			(2)
(b) Way 1	$\mathbf{a}_1 - \mathbf{a}_2 = \pm(2\mathbf{i} + 8\mathbf{j} + \mathbf{k})$	M1: Attempt to subtract position vectors	M1 A1
		A1: Correct vector $\pm(2\mathbf{i} + 8\mathbf{j} + \mathbf{k})$ (Allow as coordinates)	
	$\text{So } p = \frac{\begin{pmatrix} 2 \\ 8 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -9 \\ -12 \\ 36 \end{pmatrix}}{\sqrt{9^2 + 12^2 + 36^2}}$	Correct formula for the distance using their vectors: $\frac{ \pm(2\mathbf{i} + 8\mathbf{j} + \mathbf{k}) \cdot \mathbf{n} }{ \mathbf{n} }$	M1
	$p = \frac{\pm 78}{\sqrt{1521}} = \frac{\pm 78}{39} = 2$	M1: Correctly forms a scalar product in the numerator and Pythagoras in the denominator. (Dependent on the previous method mark)	dM1 A1
		A1: 2 (not -2)	
			(5)
(b) Way 2	$(\mathbf{i} - \mathbf{j} + \mathbf{k}) \cdot (3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}) = -13$ (d_1)	M1: Attempt scalar product between their \mathbf{n} and either position vector	M1A1
	$(3\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}) \cdot (3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}) = 13$ (d_2)	A1: Both scalar products correct	
	$\frac{\pm 13}{\sqrt{3^2 + 4^2 + 12^2}} (=1)$	Divides either of their scalar products by the magnitude of their normal vector. $\frac{d_1 \text{ or } d_2}{ \mathbf{n} }$	M1
	$p = \frac{d_1}{ \mathbf{n} } - \frac{d_2}{ \mathbf{n} } \text{ or } 2 \times \frac{d_1}{ \mathbf{n} }$	M1: Correct attempt to find the required distance i.e. subtracts their $\frac{d_1}{ \mathbf{n} }$ and $\frac{d_2}{ \mathbf{n} }$ or doubles their $\frac{d_1}{ \mathbf{n} }$ if $ d_1 = d_2 $. (Dependent on the previous method mark)	dM1 A1
		A1: 2 (not -2)	
			(5)
			Total 7

Question Number	Scheme		Marks
3. (a)			<p>A closed curve approximately symmetrical about both axes. A vertical line to the right of the curve. A horizontal line from any point on the ellipse to the vertical line with both P and N clearly marked.</p> <p>B1 (1)</p>
3. (b)	M is $\left(\frac{x+8}{2}, y\right) = (X, Y)$ or $\left(\frac{6\cos\theta+8}{2}, 3\sin\theta\right) = (X, Y)$	M1: Finds the mid-point of PN	M1A1
	$\frac{(2X-8)^2}{36} + \frac{Y^2}{9} = 1$	M1: Attempt cartesian equation A1: Correct equation	
			(4)
The next 3 marks are dependent on having the equation of a circle.			
(c)	Circle because equation may be written $(x-4)^2 + y^2 = 3^2$	Convincing argument – allow follow through provided they do have a circle! Can be implied by their centre and radius.	B1ft
	The centre is (4, 0) and the radius is 3	M1: Use their circle equation to find centre and radius A1: Correct centre and radius	M1A1
			(3)
Total 8			
<p>Special Case: In (b) they assume the locus is a circle and find the intercepts on the x-axis as (1, 0) and (7, 0) and hence deduce the centre (4, 0) and radius 3. This approach scores no marks in (b) but allow recovery in (c).</p>			

Question Number	Scheme		Marks
4.	$\begin{pmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1+s+t \\ -1+s+2t \\ 2 & -2t \end{pmatrix}$	M1: Writes Π_1 as a single vector	M1A1
		A1: Correct statement	
	$\begin{pmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1+s+t \\ -1+s+2t \\ 2 & -2t \end{pmatrix} = \begin{pmatrix} 2+2s+2t+6-6t \\ -2+2s+4t-2+2t \\ -1+s+2t+4-4t \end{pmatrix}$		M1A1
	M1: Correct attempt to multiply A1: Correct vector in any form		
	$= \begin{pmatrix} 8+2s-4t \\ -4+2s+6t \\ 3+s-2t \end{pmatrix}$	Correct simplified vector	B1
	$\mathbf{r} = \begin{pmatrix} 8 \\ -4 \\ 3 \end{pmatrix} + s \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} -4 \\ 6 \\ -2 \end{pmatrix}$		
	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & 1 \\ -4 & 6 & -2 \end{vmatrix} = -10\mathbf{i} + 20\mathbf{k}$	M1: Attempts cross product of their direction vectors	M1A1
		A1: Any multiple of $-10\mathbf{i} + 20\mathbf{k}$	
$(8\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}) \cdot (\mathbf{i} - 2\mathbf{k}) = 8 - 6$	Attempt scalar product of their normal vector with their position vector	M1	
$\mathbf{r} \cdot (\mathbf{i} - 2\mathbf{k}) = 2$	Correct equation (accept any correct equivalent e.g. $\mathbf{r} \cdot (-10\mathbf{i} + 20\mathbf{k}) = -20$)	A1	
		(9)	

Question Number	Scheme		Marks
5(a)	$I_n = \left[x^n (2x-1)^{\frac{1}{2}} \right]_1^5 - \int_1^5 nx^{n-1} (2x-1)^{\frac{1}{2}} dx$	M1: Parts in the correct direction including a valid attempt to integrate $(2x-1)^{-\frac{1}{2}}$	M1 A1
		A1: Fully correct application – may be un-simplified. (Ignore limits)	
	$I_n = \underline{5^n \times 3 - 1} - \int_1^5 nx^{n-1} \underline{(2x-1)(2x-1)^{-\frac{1}{2}}} dx$	Obtains a correct (possibly un-simplified) expression using the limits 5 and 1 and writes $(2x-1)^{\frac{1}{2}}$ as $(2x-1)(2x-1)^{-\frac{1}{2}}$	B1
	$I_n = 5^n \times 3 - 1 - 2nI_n + nI_{n-1}$	Replaces $\int x^n (2x-1)^{-\frac{1}{2}} dx$ with I_n and $\int x^{n-1} (2x-1)^{-\frac{1}{2}} dx$ with I_{n-1}	dM1
$(2n+1)I_n = nI_{n-1} + 3 \times 5^n - 1 *$	Correct completion to printed answer with no errors seen	A1cso	
			(5)
(b)	$I_0 = \int_1^5 (2x-1)^{-\frac{1}{2}} dx = \left[(2x-1)^{\frac{1}{2}} \right]_1^5 = 2$	$I_0 = 2$	B1
	$5I_2 = 2I_1 + 74 \text{ and } 3I_1 = I_0 + 14$	M1: Correctly applies the given reduction formula twice	M1 A1
		A1: Correct <u>equations</u> for I_2 and I_1 (may be implied)	
	So $I_1 = \frac{16}{3}$ and $I_2 = \dots$ or $5I_2 = 2 \frac{I_0 + 14}{3} + 74 \text{ and } I_2 = \dots$	Completes to obtains a numerical expression for I_2	dM1
	$I_2 = \frac{254}{15}$		B1
			(5)
			Total 10

Question Number	Scheme		Marks
6. (a)	$\begin{pmatrix} 4 & 2 & 3 \\ 2 & b & 0 \\ a & 1 & 8 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ \dots \\ \dots \end{pmatrix}, = \lambda \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \lambda = 8$	M1: Multiplies the given matrix by the given eigenvector	M1, M1, A1
		M1: Equates the x value to λ	
		A1: $\lambda = 8$	
			(3)
(b)	$\begin{pmatrix} 8 \\ 2+2b \\ a+2 \end{pmatrix} = "8" \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \text{ So } a = -2 \text{ and } b = 7$	M1: Their $2 + 2b = 2\lambda$ or their $a + 2 = 0$	M1 A1 A1
		A1: $b = 7$ or $a = -2$	
		A1: $b = 7$ and $a = -2$	
			(3)
(c)	$\begin{vmatrix} 4-\lambda & 2 & 3 \\ 2 & 7-\lambda & 0 \\ -2 & 1 & 8-\lambda \end{vmatrix} = 0$		M1
	$\therefore (4-\lambda)(7-\lambda)(8-\lambda) - 2 \times 2(8-\lambda) + 3(2+2(7-\lambda)) = 0$		
	<p>Correct attempt to establish the Characteristic Equation. = 0 is required but may be implied by later work Allow this mark if the equation is in terms of a, b, c</p>		
	<p>Attempts to factorise i.e. $(8-\lambda)(30-11\lambda+\lambda^2)$ or $(6-\lambda)(40-13\lambda+\lambda^2)$ or $(5-\lambda)(48-14\lambda+\lambda^2)$ (NB $240-118\lambda+19\lambda^2-\lambda^3=0$)</p>		M1 A1
	<p>M1: Attempt to factorise their cubic – an attempt to identify a linear factor and processes to obtain a simplified quadratic factor A1: Correct factorisation into one linear and one quadratic factor</p>		
	<p>Eigenvalues are 5 and 6</p>		M1: Solves their equation to obtain the other eigenvalues A1: 5 and 6
			(5)
			Total 8

Question Number	Scheme		Marks
7(a)	Put $6\cosh x = 9 - 2\sinh x$		M1
	$6 \times \frac{1}{2}(e^x + e^{-x}) = 9 - 2 \times \frac{1}{2}(e^x - e^{-x})$	Replaces $\cosh x$ and $\sinh x$ by the correct exponential forms	M1
	$4e^x + 2e^{-x} - 9 = 0 \Rightarrow 4e^{2x} - 9e^x + 2 = 0$	M1: Multiplies by e^x A1: Correct quadratic in e^x in any form with terms collected	M1 A1
	So $e^x = \frac{1}{4}$ or 2 and $x = \ln 2$ or $\ln \frac{1}{4}$	M1: Solves their quadratic in e^x A1: Correct values of x (Any correct equivalent form)	M1 A1
			(6)
(b)	Area is $\int (9 - 2\sinh x - 6\cosh x) dx$	$\int (9 - 2\sinh x - 6\cosh x) dx$ or $\int (6\cosh x - (9 - 2\sinh x)) dx$ or the equivalent in exponential form	M1
	$\pm(9x - 2\cosh x - 6\sinh x)$ or $\pm(9x - 4e^x + 2e^{-x})$	M1: Attempt to integrate A1: Correct integration	M1 A1
	$\pm\left(9\ln 2 - 2\cosh \ln 2 - 6\sinh \ln 2\right) - \left(9\ln \frac{1}{4} - 2\cosh \ln \frac{1}{4} - 6\sinh \ln \frac{1}{4}\right)$		dM1
	Complete substitution of their limits from part (a). Depends on both previous M's		
	$= \pm\left(9\ln\left(2 \div \frac{1}{4}\right) - (e^{\ln 2} + e^{-\ln 2}) - 3(e^{\ln 2} - e^{-\ln 2}) + (e^{\ln \frac{1}{4}} + e^{-\ln \frac{1}{4}}) + 3(e^{\ln \frac{1}{4}} - e^{-\ln \frac{1}{4}})\right)$		M1
	Combines logs correctly and uses cosh and sinh of ln correctly at least once		
	$\left(9\ln 8 - \frac{5}{2} - \frac{18}{4} + 4.25 - 11.25\right) = 9\ln 8 - 14$ or $27\ln 2 - 14$ Any correct equivalent		A1cao
Subtracting the wrong way round could score 5/6 max			
		(6)	
		Total 12	
	Note If they use $4e^{2x} - 9e^x + 2$ in (b) to find the area – no marks		

Question Number	Scheme		Marks
8(a)	$\frac{dy}{dx} = x^{-\frac{1}{2}}$	Correct derivative (may be unsimplified)	B1
	$s = \int \sqrt{1+(x^{-\frac{1}{2}})^2} dx = \int_1^8 \sqrt{1+\frac{1}{x}} dx$	A correct formula quoted or implied. There must be some working before the printed answer.	B1
			(2)
(b)	$x = \sinh^2 u \Rightarrow \frac{dx}{du} = 2 \sinh u \cosh u$	Correct derivative	B1
	$(1 + \frac{1}{x}) = 1 + \operatorname{cosech}^2 u = \coth^2 u$	$(1 + \frac{1}{x}) = \coth^2 u$ or $(1 + \frac{1}{x}) = \frac{\cosh^2 u}{\sinh^2 u}$ (may be implied by later work)	B1
	$s = \int \coth u \cdot 2 \sinh u \cosh u du = \int 2 \cosh^2 u du$	M1: Complete substitution A1: $\int 2 \cosh^2 u du$	M1 A1
	$= u + \frac{1}{2} \sinh 2u$ or $\frac{1}{4} e^{2u} + u - \frac{1}{4} e^{-2u}$	M1: Uses $\cosh 2u = \pm 2 \cosh^2 u \pm 1$ or changes to exponentials in an attempt to integrate an expression of the form $k \cosh^2 u$ A1: Correct integration	dM1 A1
	$x = 8 \Rightarrow u = \operatorname{arsinh} \sqrt{8} = \ln(3 + 2\sqrt{2}), x = 1 \Rightarrow u = \operatorname{arsinh} 1 = \ln(1 + \sqrt{2})$		
	$\left[u + \frac{1}{2} \sinh 2u \right]_{\operatorname{arsinh} 1}^{\operatorname{arsinh} \sqrt{8}}$ $= \operatorname{arsinh} \sqrt{8} + \frac{1}{2} \sinh(2 \operatorname{arsinh} \sqrt{8}) - (\operatorname{arsinh} 1 + \frac{1}{2} \sinh(2 \operatorname{arsinh} 1))$ <p>or</p> $\left[\frac{1}{4} e^{2u} + u - \frac{1}{4} e^{-2u} \right]_{\operatorname{arsinh} 1}^{\operatorname{arsinh} \sqrt{8}}$ $= \frac{1}{4} e^{\operatorname{arsinh} \sqrt{8}} + \operatorname{arsinh} \sqrt{8} - \frac{1}{4} e^{-2 \operatorname{arsinh} 1}$ <p>or</p> $\left[\operatorname{arsinh} \sqrt{x} + \frac{1}{2} \sinh(2 \operatorname{arsinh} \sqrt{x}) \right]_1^8$ $= \operatorname{arsinh} \sqrt{8} + \frac{1}{2} \sinh(2 \operatorname{arsinh} \sqrt{8}) - (\operatorname{arsinh} 1 + \frac{1}{2} \sinh(2 \operatorname{arsinh} 1))$		ddM1A1
	M1: The limits $\operatorname{arsinh} \sqrt{8}$ and $\operatorname{arsinh} 1$ or their $\ln(3 + 2\sqrt{2})$ and $\ln(1 + \sqrt{2})$ used correctly in their $f(u)$ or the limits 8 and 1 used correctly if they revert to x Dependent on both previous M's A1: A completely correct expression		
	$\ln(1 + \sqrt{2}) + 5\sqrt{2}$		A1
			(9)
			Total 11

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